Name: Solutions

Date: 5/5/2025

Math 130
Exam 4

Please show ALL your work on the problems below. No more than 1 point will be given to problems if you only provide the correct answer and insufficient work.

1. (22 points) To illustrate the effects of driving under the influence (DUI) of alcohol, a police officer brought a DUI simulator to a local high school. Student reaction time in an emergency was measured with unimpaired vision and also while wearing a pair of special goggles to simulate the effects of alcohol on vision. For a random sample of six teenagers, the time (in seconds) required to bring the vehicle to a stop from a speed of 60 miles per hour was recorded. 2-pop average problem/Dependent samples

Subject	1	2	3	4	5	6
Braking Time With Normal Vision (seconds)	4.4	4	5.3	5.7	4.8	4.1
Braking Time With Impaired Vision (seconds)	5.7	5.8	5.1	6.5	5.7	5.8
Differences (Impaired − Normal)	1.3	1.8	−0.2	0.8	0.9	1.7

Test the claim that braking time is longer with impaired vision than with normal vision at the 0.02 significance level. Use the rejection region method.

Hyp. Test

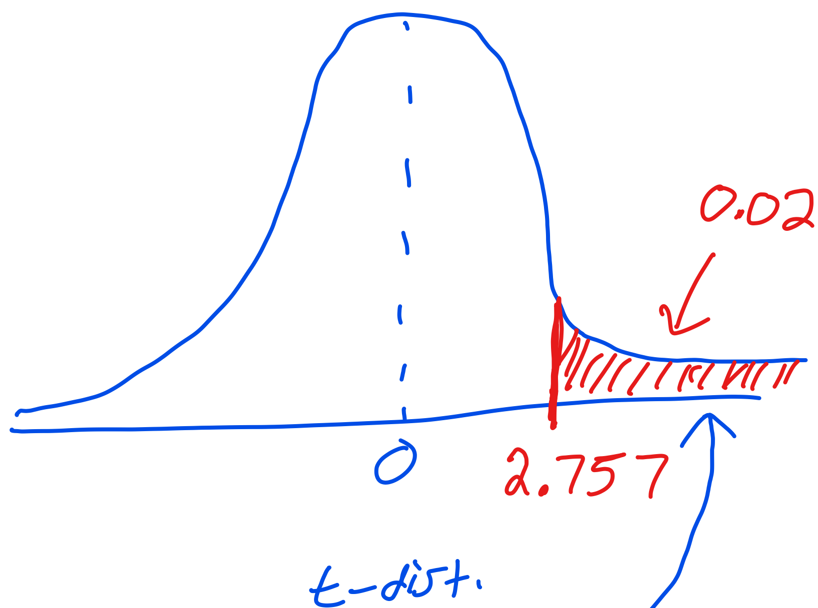
$H_0: \mu_d = 0$

$H_1: \mu_d > 0$

μ_d = The average of all differences in braking time (Impaired time − Normal time)

Rejection Region

$\alpha = 0.02$ $\quad df = n-1 = 6-1 = 5$



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Test Stat

$n = 6$

$$\bar{d} = \frac{1.3 + 1.8 + \cdots + 1.7}{6} = 1.05$$

$$\Sigma x^2 = 1.3^2 + 1.8^2 + \cdots + 1.7^2 = 9.31$$

$$\Sigma x = 1.3 + 1.8 + \cdots + 1.7 = 6.3$$

$$s_d = \sqrt{\frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1}} = \sqrt{\frac{9.31 - \frac{(6.3)^2}{6}}{6-1}} = 0.7341661937$$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} = \frac{1.05 - 0}{\frac{0.7341661937}{\sqrt{6}}} = 3.503245249$$

Conclusion Reject H_0!

Evidence suggests that braking time is longer (on average) with impaired vision than for normal vision.

1-pop average problem

2. (22 points) Some people have said that a college education is not as important as it once was. One way to test this is to look at how having a college degree effects a person's salary. As of today, the average salary of all people in California is \$73,220 per year. Do California employees with a college degree make the same? To test this, 100 California employees with a college degree were polled and their salaries had a mean of \$91,380 and a standard deviation of \$3920. Test this claim at the 0.10 significance level. Use the rejection region method.

Hyp. Test

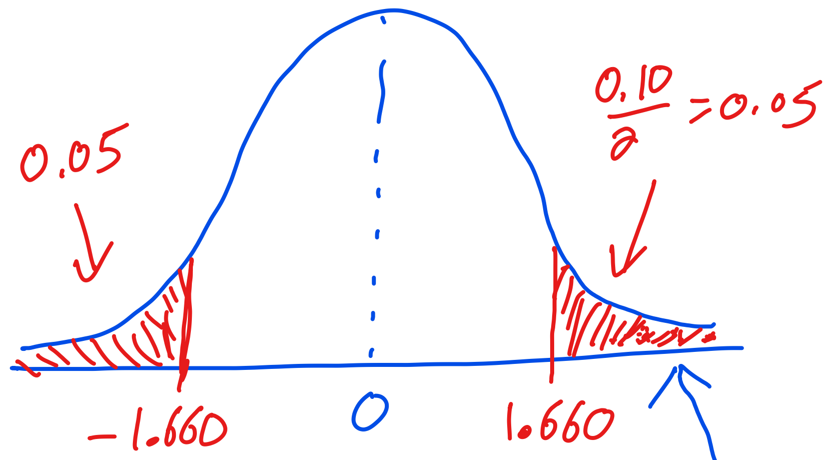
$H_0: \mu = \$73,220$

$H_1: \mu \neq \$73,220$

μ = The average salary of all California employees that have a college degree

Rejection Region

$\alpha = 0.10 \qquad df = n-1 = 100-1 = 99$ (use 100)



t-dist.

Test Stat

$n = 100$

$\bar{x} = \$91,380$

$s = \$3920$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{91380 - 73220}{\frac{3920}{\sqrt{100}}} = 46.3265306$$

Conclusion

Reject H_0!

Evidence suggests that California employees with a college degree do not make the same (on average) than California employees without a college degree.

2-pop percent problem/Independent samples

3. (22 points) Researchers wondered if there was a difference between males and females in regard to some common annoyances. They asked a random sample of males and females, the following question: "Are you annoyed by people who repeatedly check their mobile phones while having an in-person conversation?" Among the 526 males surveyed, 198 responded "Yes". Among the 543 females surveyed, 216 responded "Yes." Does the evidence suggest a higher proportion of females are annoyed by this behavior at the 0.08 significance level? Use the p-value method.

pop 1 = All males

p_1 = The percentage of all males that get annoyed when a person is constantly checking their mobile phone while having an in-person conversation

pop 2 = All females

p_2 = The percentage of all females that get annoyed when a person is constantly checking their mobile phone while having an in-person conversation

Sample 1

$n_1 = 526 \quad x_1 = 198 \quad \hat{p}_1 = \frac{x_1}{n_1} = \frac{198}{526}$

Sample 2

$n_2 = 543 \quad x_2 = 216 \quad \hat{p}_2 = \frac{x_2}{n_2} = \frac{216}{543}$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{198 + 216}{526 + 543} = \frac{414}{1069}$$

$$\hat{q} = 1 - \hat{p} = \frac{1069}{1069} - \frac{414}{1069} = \frac{655}{1069}$$

Hyp. Test

$H_0: p_1 = p_2$

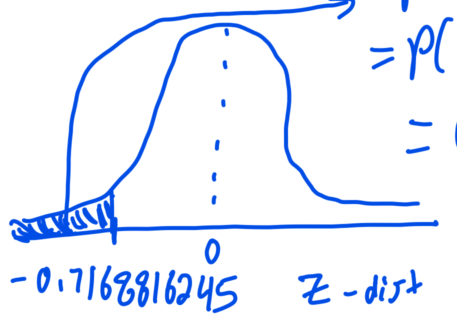
$H_1: p_1 < p_2$

Test stat

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\left(\frac{198}{526} - \frac{216}{543}\right) - (0)}{\sqrt{\frac{414}{1069} \cdot \frac{655}{1069}}\sqrt{\frac{1}{526} + \frac{1}{543}}}$$

$$z = -0.7168816245$$

p-value $\quad \alpha = 0.08$



p-value $= P(z < -0.7168816245)$

$= 0.2367$

Is p-value $< \alpha$?

$0.2367 < 0.08$? No!

Conclusion: Do not reject H_0!

Not enough evidence to say that a higher proportion of females are annoyed by this behavior.

1-pop σ problem

4. (22 points) A doctor says that the standard deviation of the lengths of stay for patients involved in a crash in which the vehicle struck a tree is 6.14 days. A random sample of 30 lengths of stay for patients involved in this type of crash has a standard deviation of 5.8 days. At the $\alpha = 0.05$ level of significance, can you reject the doctor's claim? Use the p-value method.

Hyp. Test

$H_0: \sigma = 6.14$ days

$H_1: \sigma \neq 6.14$ days

σ = The standard deviation for the lengths of stay for all patients involved in a crash where the vehicle struck a tree

Test stat

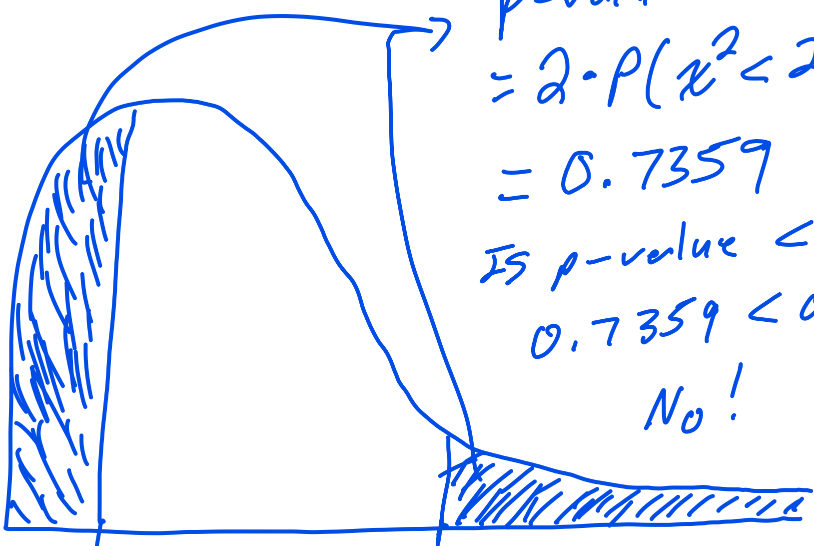
$n = 30 \qquad s = 5.8$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(30-1)(5.8)^2}{(6.14)^2}$$

$$= 25.877197636$$

p-value

$\alpha = 0.05 \qquad df = n-1 = 30-1 = 29$



(bec. $s < \sigma$
$5.8 < 6.14$)

χ^2-dist.

p-value

$= 2 \cdot P(\chi^2 < 25.877197636)$

$= 0.7359$

Is p-value $< \alpha$?

$0.7359 < 0.05$?

No!

Conclusion

Do not reject H_0!

Not enough evidence to reject the doctor's claim.

1-pop percentage problem

5. (22 points) In 2015, the proportion of California residents who lived in an apartment was 19.3%. To see if this proportion has dropped since then, 380 California residents were randomly selected and 64 said that they currently live in an apartment. Test the claim at the 0.04 level of significance. Use the p-value method.

Hyp. Test

$H_0: p = 19.3\%$

$H_1: p < 19.3\%$

p = The percentage of of all California residents who currently live in an apartment

Test Stat

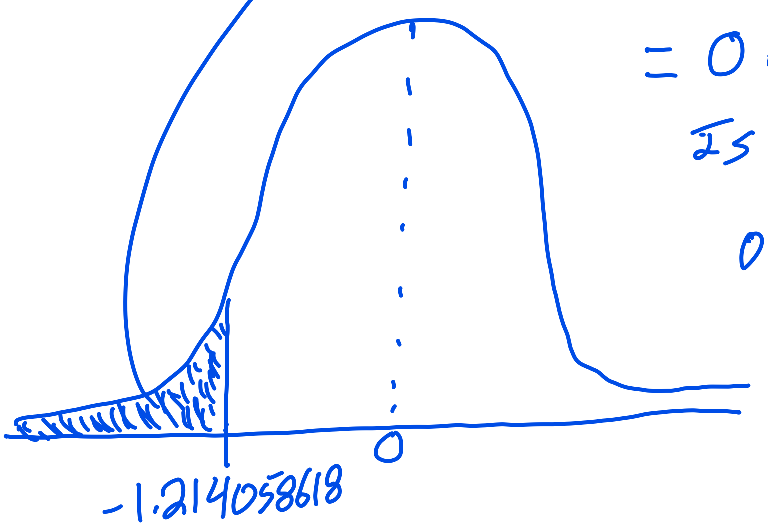
$n = 380 \quad x = 64 \quad \hat{p} = \frac{x}{n} = \frac{64}{380}$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{64}{380} - 0.193}{\sqrt{\frac{(0.193)(0.807)}{380}}}$$

$$= -1.214058618$$

p-value

$\alpha = 0.04$



p-value

$= P(z < -1.214058618)$

$= 0.11236$

Is p-value $< \alpha$?

$0.11236 < 0.04$? No!

Conclusion Do not reject H_0!

Not enough evidence to say that the percentage of all California residence who currently live in an apartment has dropped since 2015.

6. (7 points) What does a the 0.03 mean in a hypothesis test that is performed at the 0.03 significance level.

If you perform the same hypothesis test many times, each time with a new sample, you will reject Ho when Ho is true about 3 % of the time.

7. (3 points) What is a type II error?

when you do not reject Ho when Ho is false

Some formulas you may need:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \qquad df = n - 1$$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad df = n - 1$$

$$z = \frac{\hat{p}_1 - \hat{p}_2 - (p_1 - p_2)}{\sqrt{\hat{p}\hat{q}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad \hat{p} = \frac{x_1 + x_2}{n_1 + n_2}$$

$$t = \frac{\bar{d} - \mu_d}{\frac{s_d}{\sqrt{n}}} \qquad t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \qquad df = \text{smaller of } n_1 - 1 \ \& \ n_2 - 1$$

$$F = \frac{s_1^2}{s_2^2} \qquad df_1 = n_1 - 1 \qquad df_2 = n_2 - 1$$

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1}}$$